

A Computational Study of Base- b Deletable Primes

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Abstract

A prime is *deletable* in base b if its base- b digits can be removed one at a time, never creating a leading zero, so that every intermediate number is prime, ending at a single-digit prime. We carry out a systematic computation of the counts $D_b(n)$ of n -digit base- b deletable primes for $3 \leq b \leq 12$, contributing many new terms to ten OEIS sequences that had stalled at brute-force depth. The counts grow geometrically, and the per-step growth ratio splits sharply by the parity of b : even bases grow about twice as fast as the odd base just below them. We explain this by factoring the ratio into a mean offspring count and a mean number of deletable parents; the offspring count is governed by $b^2/(\varphi(b) \ln b)$, so the parity dichotomy comes entirely from $\varphi(b)$. We also prove two elementary obstruction theorems on which single-digit roots a deletable prime can reach, one modulo 2 (all odd bases) and one modulo 3 (bases $b \equiv 1 \pmod{3}$), generalizing an observation of Andersen for base 10), and we show by explicit counterexamples that the analogue fails for every modulus $q \geq 5$. Finally we record several new integer sequences. All code and verification records are public.

1 Introduction

Let $b \geq 2$ be an integer base. Following Caldwell [1], a prime p is a *deletable prime* in base b if either $p < b$ (a single-digit prime), or one digit can be removed from the base- b representation of p , without creating a leading zero, to give a smaller number that is again a deletable prime in base b . Reading this from the top down, a deletable prime is one that can be reduced to a single-digit prime by deleting digits one at a time, with every intermediate value prime. For example, in base 10 the prime 410256793 is deletable, witnessed by the chain

$$\begin{aligned} 410256793 &\rightarrow 41256793 \rightarrow 4125673 \rightarrow 415673 \rightarrow 45673 \\ &\rightarrow 4567 \rightarrow 467 \rightarrow 67 \rightarrow 7. \end{aligned}$$

Deletable primes sit among a family of digit-manipulation prime sets that are easy to confuse, so we fix the distinctions at the outset. A *truncatable* prime stays prime as digits are removed from one end only [2]. A *minimal* prime is one with no proper prime subword under deletion, the opposite extremal direction from the deletable primes [3, 4]. A *delicate* prime becomes composite under every single-digit change [5]. The deletable primes are the most permissive of these: a digit may be removed from any interior position, in any order. Where truncatable and minimal primes are known to be finite in every base, the deletable primes are conjectured to be infinite in every base, and no proof is known.

Our starting point is that the counting sequences for deletable primes are computationally underdeveloped. The Online Encyclopedia of Integer Sequences records $D_b(n)$, the number of n -digit base- b deletable primes, as sequences A096236 through A096245 for $3 \leq b \leq 12$ [9], and each

was flagged as hard and had stalled at the depth reachable by naive search. (The base-2 member of the family, A096235, requires a separate convention because base 2 has no single-digit prime root; we omit it here.) There is, to our knowledge, no published study of the size, density, or growth of these counts in any base. This paper fills that gap. Our contributions are:

1. a verified extension of all ten count sequences (Section 3);
2. a growth law: the counts grow geometrically with a per-step ratio that splits by the parity of b , together with a heuristic that explains the split (Section 4);
3. two elementary, verified obstruction theorems on reachable single-digit roots, and a sharp negative result (Section 5);
4. several new integer sequences (Section 6).

2 Definitions and the deletion graph

We work throughout with the strict, no-leading-zero convention.

Definition 1. Fix a base $b \geq 2$. The set $\mathcal{D}_b$ of *base- b deletable primes* is the smallest set of primes such that (i) every prime $p < b$ is in $\mathcal{D}_b$, and (ii) a prime $p \geq b$ is in $\mathcal{D}_b$ if and only if deleting some single digit of its base- b representation, with the result not beginning with the digit 0, yields a member of $\mathcal{D}_b$. We write $D_b(n) = \#\{p \in \mathcal{D}_b : p \text{ has exactly } n \text{ base-}b \text{ digits}\}$.

A looser convention, in which deletions may create leading zeros, gives a different set (OEIS A080608 for base 10); the strict convention used here is OEIS A305352 for base 10, and is the one matching the count sequences A096236–A096245. The two conventions already differ at four digits in base 10, where the strict count is 585 and the loose count is 598.

It is convenient to view $\mathcal{D}_b$ as a directed acyclic graph: the vertices are the deletable primes, and we draw an edge $p \rightarrow q$ whenever q is obtained from p by deleting one digit (without a leading zero) and q is prime. A *root* is a single-digit prime. A vertex is deletable precisely when it has a directed path to a root. This graph view drives both the computation in Section 3 and the growth analysis in Section 4.

3 Computation and verification

3.1 Forward construction

We build $\mathcal{D}_b$ one digit-length at a time. The single-digit deletable primes are the primes below b . Given the set of all $(n - 1)$ -digit deletable primes, we form n -digit candidates by inserting a single digit into each member, in every position that does not create a leading zero, and we keep those candidates that are prime. Every prime obtained this way is automatically deletable, because deleting the inserted digit returns the parent, which is deletable by construction. This forward pass is far cheaper than enumerating all n -digit primes and testing each for deletability, because the deletable primes are sparse: the candidate pool at level n has size on the order of b times the size of level $n - 1$, not b^n .

Each $(n - 1)$ -digit deletable prime Q generates exactly $nb - 1$ in-range insertion candidates: there are n positions at which a digit may be inserted and b digits to choose from, and the one combination that places the digit 0 in the leading position is excluded. Candidates are tested for primality on a GPU using a deterministic Miller-Rabin routine valid for inputs below 2^{63} , with a

small-prime screen applied first; the largest level we reach lies below this bound. Level sets are held sorted and deduplicated, since one n -digit deletable prime can arise from several distinct parents.

3.2 Extent and verification

Table 1 summarizes the ten extended sequences. We added many new terms to each; the deepest counts are on the order of several billion deletable primes at a single digit length.

Table 1: The ten count sequences after this work. “Terms” is the number of known values of $D_b(n)$; “ $D_b(n)$ at largest n ” is the deepest count reached.

base b	OEIS	terms	$D_b(n)$ at largest n
3	A096236	35	3 154 049 796
4	A096237	20	4 994 132 480
5	A096238	24	4 734 931 505
6	A096239	15	8 140 250 340
7	A096240	19	3 343 165 343
8	A096241	14	6 756 349 166
9	A096242	15	2 203 389 725
10	A096243	12	5 018 716 664
11	A096244	15	2 247 889 251
12	A096245	11	8 060 890 618

As an example, the full base-10 sequence (A096243) runs 4, 16, 94, 585, 3788, 25768, 182762, 1340905, 10135727, 78580647, 622188500, and 5018716664.

Verification follows the discipline of our earlier OEIS work. For each base we first reproduce every previously published term before recording any new one; we re-derive each level a second time with perturbed parameters (different batch sizes and memory layouts); and we cross-check a sample of every level against an independent primality engine. A natural external check also arose during the work: an independent contributor, C. Corti, deposited base-9 counts on Zenodo [8] whose values agree exactly with ours, an independent confirmation of the pipeline at that scale. The full code, unit tests, and per-level records are public (Section 7); the new terms are credited to the author in OEIS.

4 Growth of the counts

4.1 The parity dichotomy

The counts grow geometrically. Table 2 reports the deepest available value of the ratio $r_b(n) = D_b(n)/D_b(n-1)$ for each base, alongside the two simplest candidate limits, $b-1$ and $b/\ln b$.

The striking feature is the parity zigzag, plotted in Figure 1: each even base grows markedly faster than the odd base just below it ($r_8 = 6.10$ against $r_7 = 3.55$, $r_{10} = 8.07$ against $r_9 = 4.64$, and so on). This even-over-adjacent-odd comparison is robust across the whole range, though the odd bases are not themselves monotonic ($r_{11} = 4.48 < r_9 = 4.64$), a finite-size effect at the depths reached. Neither $b-1$ nor $b/\ln b$ fits across all bases, and since the ratios are still rising with n we do not claim a closed-form limit. The phenomenon to be explained is the parity split itself.

Table 2: Deepest computed growth ratio $r_b = D_b(n)/D_b(n-1)$ by base, with two candidate limits. The ratios are still slowly increasing with n .

b	parity	r_b (deepest n)	$b-1$	$b/\ln b$
3	odd	1.99	2	2.73
4	even	3.64	3	2.89
5	odd	2.78	4	3.11
6	even	5.53	5	3.35
7	odd	3.55	6	3.60
8	even	6.10	7	3.85
9	odd	4.64	8	4.10
10	even	8.07	9	4.34
11	odd	4.48	10	4.59
12	even	10.11	11	4.83

4.2 A decomposition of the ratio

The growth ratio factors exactly through the deletion graph. For a level- $(n-1)$ deletable prime Q , call its prime insertion-children its *offspring*; every offspring is deletable. Counting the edges between levels $n-1$ and n in two ways gives the identity

$$r_b(n) = \frac{D_b(n)}{D_b(n-1)} = \frac{M_b(n-1)}{I_b(n)}, \quad (1)$$

where $M_b(n-1)$ is the mean number of prime insertion-children of a level- $(n-1)$ deletable prime and $I_b(n)$ is the mean number of single-digit deletions of a level- n deletable prime that land on a level- $(n-1)$ deletable prime. Both are counted with multiplicity, by edges of the deletion graph: an insertion (equivalently, a deletion) at a different position is a separate edge even when it yields the same number. Writing E for the number of edges between the two levels, $M_b(n-1) = E/D_b(n-1)$ and $I_b(n) = E/D_b(n)$, so (1) is an identity. The factor $I_b(n)$ records how often a single deletable prime is reached from more than one parent, the overcounting that a pure offspring count would incur.

Each factor has a clean reading. A level- $(n-1)$ deletable prime produces about nb insertion candidates, and almost all of them inherit the parent's last digit, which is coprime to b (the parent, a multi-digit prime, ends in a coprime digit). Among n -digit numbers lying in a fixed coprime residue class modulo b , primes occur with density about $b/(\varphi(b)n \ln b)$, the prime number theorem for arithmetic progressions supplying the factor $1/\varphi(b)$. The factors of n cancel, predicting

$$M_b \approx \frac{b^2}{\varphi(b) \ln b}. \quad (2)$$

The functional form is not new in itself. Angell and Godwin [2] proposed the same shape heuristically for the maximal length of a base- b left-truncatable prime, $\sim b^2 e/(\varphi(b) \ln b)$ (restated in [6]), and in both settings it comes from the prime number theorem in arithmetic progressions applied to a fixed coprime residue class. What is specific here is the quantity it governs, the mean offspring of a deletable prime.

Table 3 compares the measured offspring mean to (2) and reports the measured parent count I_b . The offspring prediction is accurate to within about five percent for most bases; the smallest bases run higher (base 3 by roughly twenty percent), reflecting the slow finite-size convergence expected

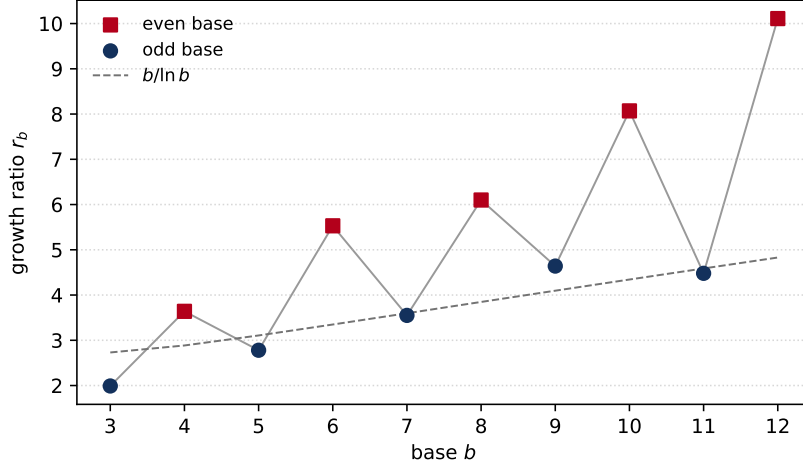


Figure 1: The deepest computed growth ratio r_b against base b . Even bases (squares) sit well above the odd bases (circles) on either side, the parity zigzag. The dashed curve is the parity-blind heuristic $b/\ln b$, which the even bases exceed.

from the prime number theorem, the same effect that keeps the growth ratios rising. The parent count I_b converges to a modest base-dependent constant for which we do not have a closed form. Because Table 2 is evaluated at the deepest level reached in OEIS while Table 3 is evaluated at the deepest level we enumerate completely, the two are at different n ; the quotient M_b/I_b in Table 3 is the growth ratio at that shallower level, and the identity (1) holds at each fixed n .

Table 3: The decomposition (1), measured from the deletion graph at the deepest digit length we enumerate completely. M_b is the offspring mean; $b^2/(\varphi(b) \ln b)$ is the prediction (2); I_b is the mean number of deletable parents (counted by edges).

b	parity	M_b (measured)	$b^2/(\varphi(b) \ln b)$	I_b (measured)
3	odd	5.04	4.10	2.64
4	even	5.98	5.77	1.82
5	odd	4.16	3.88	1.68
6	even	9.66	10.05	1.96
7	odd	4.88	4.20	1.62
8	even	7.31	7.69	1.38
9	odd	6.32	6.14	1.56
10	even	10.63	10.86	1.50
11	odd	5.32	5.05	1.33
12	even	13.42	14.49	1.56

This isolates the source of the parity dichotomy. The offspring mean M_b carries the factor $1/\varphi(b)$, and for even b the totient $\varphi(b)$ is at most $b/2$ (the factor of 2 in b removes half of the residues), whereas for odd b it is larger relative to b . So even bases have systematically more offspring per deletable prime, and grow faster. The parent-count factor I_b is a milder correction. We summarize the empirical situation as a conjecture.

Conjecture 1. For each fixed base b the ratio $r_b(n)$ tends to a finite limit $r_b > 1$ as $n \rightarrow \infty$, the

counts $D_b(n)$ grow geometrically, and the limits satisfy $r_b = M_b/I_b$ with $M_b \rightarrow b^2/(\varphi(b) \ln b)$. In particular r_b is larger for even b than for the neighboring odd bases.

We do not claim a closed form for the limit itself, since the parent count I_b is at present only measured.

5 Obstructions on reachable roots

A deletable prime may reach several single-digit roots through different deletion chains. We record which roots can co-occur. The arguments rest on a single fact: if $b \equiv 1 \pmod{q}$ then $b^i \equiv 1 \pmod{q}$ for all i , so every integer is congruent to its digit sum modulo q .

Theorem 1 (Odd bases). *Let b be odd. A base- b deletable prime reaches at most one odd single-digit prime root. (The root 2 may also be reachable and is not restricted.)*

Proof. Since b is odd, every integer is congruent to its digit sum modulo 2. Consider a deletion chain ending at an odd root. Every member of the chain is a prime greater than 2, hence odd, hence of odd digit sum. Deleting a digit d changes the digit sum by d , so to keep the digit sum odd the deleted digit d must be even. Thus along any chain ending at an odd root, no odd digit is ever deleted. Consequently every odd digit of the starting prime survives all the way to the single remaining digit, which forces the starting prime to contain exactly one odd digit. That digit is then the unique odd root. $\square$

Theorem 2 (Bases $b \equiv 1 \pmod{3}$). *Let $b \equiv 1 \pmod{3}$. All single-digit prime roots reachable from a given base- b deletable prime, other than the root 3, lie in a single residue class modulo 3.*

Proof. Since $b \equiv 1 \pmod{3}$, every integer is congruent to its digit sum modulo 3. Let p be a deletable prime and consider any chain from p to a root $r \not\equiv 0 \pmod{3}$. Every member of the chain is a prime; a prime member exceeding 3 is not divisible by 3, so its residue modulo 3 lies in $\{1, 2\}$. Deleting a digit d changes the residue by $-d \pmod{3}$. From residue 1 one cannot delete a digit $\equiv 1$ and from residue 2 one cannot delete a digit $\equiv 2$, since either would produce a multiple of 3, which is not prime above 3. Hence each deletion of a digit $\not\equiv 0 \pmod{3}$ flips the residue between 1 and 2, while deleting a digit $\equiv 0 \pmod{3}$ leaves it unchanged. To reach the single-digit root r , one deletes every digit of p except r ; the deleted digits that are $\not\equiv 0 \pmod{3}$ number exactly $m - 1$, where m is the number of digits of p that are $\not\equiv 0 \pmod{3}$ (the surviving digit r is one of these m). The residue is therefore flipped exactly $m - 1$ times along every such chain, so the residue of r equals the residue of p flipped $m - 1$ times, a value that does not depend on the chain. Thus all reachable roots $\not\equiv 0 \pmod{3}$ share one residue class modulo 3. $\square$

Theorem 2 generalizes an observation of Andersen [7], who showed for base 10 that no deletable prime reaches both 7 and 2, or both 7 and 5; here $7 \equiv 1$ and $2, 5 \equiv 2 \pmod{3}$. The two theorems share a frame: deleting a digit shifts the residue modulo q , and avoiding a multiple of q constrains the shifts. Modulo 2 there is only one nonzero class, so a shift to the other nonzero class is impossible and no digit of that class can ever be deleted, which is why Theorem 1 pins a unique root rather than a class. Modulo 3 there are two nonzero classes and shifts between them are allowed, so only the class of the root is determined.

The picture is sharp: it does not extend.

Proposition 1 (Sharpness). *For every modulus $q \geq 5$ the analogue of Theorem 2 is false: there exist bases $b \equiv 1 \pmod{q}$ and deletable primes reaching roots in two different nonzero residue*

classes modulo q . For instance, in base 6 the deletable prime written 311 in base 10 reaches the roots 2, 3, 5, which span two classes modulo 5; base 16 and base 21 furnish further counterexamples.

The reason is structural. For $q \geq 5$ there are at least four nonzero residue classes, and from a given residue a deletion can move to several of them while avoiding 0, so no parity-of-flips invariant pins the root class. We verified Theorems 1 and 2 exhaustively over all deletable primes below 2×10^6 in the relevant bases, with no exceptions, and located the counterexamples of Proposition 1 in the same range.

6 New sequences

The smallest and largest n -digit deletable prime are recorded in OEIS only for base 10 (A125589, A125590). We computed both for the odd bases $b = 3, 5, 7, 11$. Tables 4 and 5 give the values; none of these eight sequences was present in OEIS.

Table 4: Smallest n -digit deletable prime (in base 10) for $b = 3, 5, 7, 11$.

n	$b = 3$	$b = 5$	$b = 7$	$b = 11$
1	2	2	2	2
2	5	7	17	13
3	11	37	101	149
4	29	137	691	1361
5	83	677	4903	14891
6	251	3389	33851	161339
7	809	15889	235577	
8	2267	79889		
9	6803	392389		

Table 5: Largest n -digit deletable prime (in base 10) for $b = 3, 5, 7, 11$.

n	$b = 3$	$b = 5$	$b = 7$	$b = 11$
1	2	3	5	7
2	7	23	47	113
3	23	113	331	1301
4	73	613	2389	14593
5	233	3067	16729	160757
6	719	15439	117571	1771051
7	2153	77687	823153	
8	6473	389947		
9	19501	1951289		

We also record, for each base, the largest number of distinct deletion paths from an n -digit deletable prime down to a root (Table 6). This extends Huber's base-10 path-count sequence A392157 to a cross-base record family; it too is absent from OEIS for $b \neq 10$.

Table 6: Largest number of distinct deletion paths to a root among n -digit deletable primes.

n	$b = 3$	$b = 5$	$b = 7$	$b = 11$
1	1	1	1	1
2	1	2	2	2
3	2	4	4	4
4	6	8	8	8
5	12	24	16	24
6	36	96	48	48
7	108	216	120	

7 Reproducibility

All code, unit tests, and per-level verification records are released publicly at <https://github.com/OrygnsCode/deletable-primes-family>. A single base and digit range can be reproduced independently in PARI/GP: the predicate

$$\text{isdel}(b,p) = (p \text{ is a base-}b \text{ deletable prime})$$

is a short recursion over the digit deletions, and counts can be checked against standard primality libraries. We carried out such cross-checks as part of the verification in Section 3. The extended count sequences are deposited in OEIS under the A-numbers in Table 1.

8 Concluding remarks

Several questions remain open. The most basic is the infinitude of deletable primes in each base, which is conjectured but unproven and which our growth data supports indirectly: if $D_b(n)$ grows geometrically with ratio above 1, the supply does not run out. The growth law itself is open at two levels: whether $r_b(n)$ converges at all, and, if so, a closed form for the parent count I_b that would turn the decomposition (1) into a closed-form limit. A proof that the offspring mean satisfies $M_b \rightarrow b^2/(\varphi(b)\ln b)$, rather than the heuristic of Section 4, would be a natural first target, and is parallel to the first-moment results recently obtained for left-truncatable primes [6]. Finally, the obstruction results of Section 5 invite the question of what other digit-class invariants survive in special bases; Theorems 1 and 2 exhaust the clean modular case, but bases with particular factorizations may carry their own constraints.

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